- Linear momentum is defined as the product of a system's mass multiplied by its velocity.

$$\boxed{\vec{p} = m\vec{v}}$$

Units: kg m/s

Newton's Second Law

- The importance of momentum was recognized early in the development of classical physics.
  - It was called the "quantity of motion."
- Newton stated his second law of motion in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

- This statement of Newton's second law applies to all situations.
  - $F = ma$ is a special case

$$F = \frac{\Delta p}{\Delta t} \qquad \Delta p = m\Delta v$$

$$F = \frac{m\Delta v}{\Delta t} \quad a = \frac{\Delta v}{\Delta t}$$

$$F = ma$$

When the mass of the system is constant.

Example

Water leaves a hose at a rate of 1.5 kg/s with a speed of 20 m/s and is aimed at the side of a car which stops it.

a) What is the force of the water exerted on the car?

b) If the water splashes back, will the force be greater or less?

a) $\vec{F} = \frac{\Delta \vec{p}}{\Delta t} \qquad \vec{p} = m\vec{v}$

$$F = \frac{m\Delta v}{\Delta t}$$

$$F = 1.5(0 - 20) = -30 \text{ N}$$

This is the force of the car stopping the water.

$$F = 30 \text{ N}$$

b) The force will be greater since the change in velocity will be greater.

Impulse

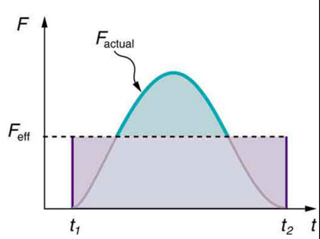
- The effect of a force on an object depends on how long it acts, as well as how great the force is.
  - A very large force acting for a short time has a great effect on the momentum of a small ball.
  - A small force could cause the same change in momentum, but it would have to act for a much longer time.

- This effect can be shown mathematically by rearranging $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$ to give

$$\boxed{\Delta \vec{p} = \vec{F} \Delta t}$$

- The quantity $\vec{F}\Delta t$ is given the name **impulse**.
  - Impulse is the same as the change in momentum.

- The definition of impulse includes an assumption that the force is constant over the time interval Δt.
- Forces usually vary considerably even during the brief time intervals considered.
- It is possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force.



Example

- A batter hits a 90 mph (40.5 m/s) baseball (m = 150 g) with an average force of 480 N. The bat is in contact with the ball for 0.017 s. Calculate the velocity of the ball off the bat.

$$\Delta\vec{p} = \vec{F}\Delta t \qquad \vec{p} = m\vec{v}$$

$$m\Delta v = m(v_f - v_i) = F\Delta t$$

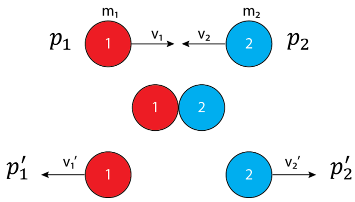
$$v_f = \frac{F\Delta t}{m} + v_i$$

$$v_f = \frac{(-480)(0.017)}{0.150} + 40.5 = -14 \text{ m/s}$$

(The baseball leaves the bat in the opposite direction.)

Conservation of Momentum

- Linear momentum is conserved.
  - The linear momentum of a system is constant.
- Shortly before Newton's time it had been observed that the vector sum of the momentum of two colliding objects remains constant.



$\Delta p_1 = F_1 \Delta t$ $\Delta p_2 = F_2 \Delta t$

Δt is the same for both balls

According to Newton's third law, $F_2 = -F_1$, so

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1$$

$$\Delta p_1 + \Delta p_2 = 0$$

The total momentum of the system is constant.

$$p_1 + p_2 = p'_1 + p'_2 = constant$$

- It can be similarly shown that total momentum is conserved for any isolated system, with any number of objects in it.
  - An isolated system is defined to be one for which the net external force is zero ($F_{net} = 0$).
  - The total momentum can be shown to be the momentum of the center of mass of the system.

Law of Conservation of Momentum

- The total momentum of any isolated system remains constant.

$$\sum \vec{p} = constant$$

or

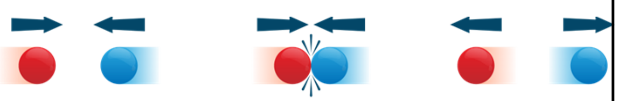
$$\sum \vec{p} = \sum \vec{p}'$$

Collisions

Marcel Langthim (Pixabay)

Elastic Collisions

- An **elastic collision** is one that conserves internal kinetic energy.
  - Internal kinetic energy is the sum of the kinetic energies of the objects in the system.



VectorMine (Adobe Stock)

Example

A marble moving to the right at 15 m/s on a frictionless surface makes an elastic head-on collision with an identical marble at rest. Calculate the velocities of the marbles after the collision.

Net force is zero, therefore momentum is conserved.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Substituting known values gives

$$5 = v_1' + v_2'$$

Elastic collision, therefore kinetic energy is conserved.

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

Substituting known values gives

$$25 = v_1'^2 + v_2'^2$$

Solve

$$5 = v_1' + v_2'$$

$$25 = v_1'^2 + v_2'^2$$

$$v_1' = 5 - v_2'$$

$$25 = (5 - v_2')^2 + v_2'^2$$

$$25 = (25 - 10v_2' + v_2'^2) + v_2'^2$$

$$0 = -10v_2' + 2v_2'^2$$

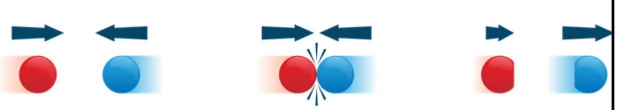
$$0 = (-10 + 2v_2')v_2'$$

$$v_2' = 5\ \mathrm{m/s}$$

Note: $v_2' = 0$ does not make physical sense as a solution, so it is discarded.

Inelastic Collisions

- An **inelastic collision** is one is which the internal kinetic enery is not conserved.



VectorMine (Adobe Stock)

Example

A 4500 kg truck traveling at 15.0 m/s east collides with a 1500 kg car initially at rest. The car and truck stick together and move together after the collision. Calculate the final velocity of the two-vehicle mass.

Net force is zero, therefore momentum is conserved.

$$m_1 v_1 + m_2 v_2 = m_{1+2} v'_{1+2}$$

Substituting known values gives

$$(4500)(15) = (4500 + 1500) v'_{1+2}$$

$$v'_{1+2} = 11 \text{ m/s}$$